

Problem 10

Use equation (1.8) to find the fractions that are equivalent to the following repeating decimals:

$$0.576923076923076923 \dots$$

Solution

Observe the repeating pattern $\dots 076923 \dots$. The 5 at the beginning is anomalous.

$$\begin{aligned} 0.576923076923076923 \dots &= 0.5 + 0.076923 + 0.000000076923 + \dots \\ &= \frac{1}{2} + \frac{76\,923}{10^6} + \frac{76\,923}{10^{12}} + \dots \\ &= \frac{1}{2} + \sum_{i=1}^{\infty} \frac{76\,923}{10^{6i}} \\ &= \frac{1}{2} + \sum_{i=1}^{\infty} \frac{76\,923}{1\,000\,000^i} \\ &= \frac{1}{2} + 76\,923 \sum_{i=1}^{\infty} \frac{1}{1\,000\,000^i} \\ &= \frac{1}{2} + 76\,923 \sum_{i=1}^{\infty} \left(\frac{1}{1\,000\,000} \right)^i \\ &= \frac{1}{2} + 76\,923 \left[-1 + \sum_{i=0}^{\infty} \left(\frac{1}{1\,000\,000} \right)^i \right] \\ &= \frac{1}{2} + 76\,923 \left[-1 + \frac{1}{1 - \left(\frac{1}{1\,000\,000} \right)} \right] \\ &= \frac{1}{2} + 76\,923 \left(-1 + \frac{1\,000\,000}{999\,999} \right) \\ &= \frac{1}{2} + 76\,923 \left(\frac{1}{999\,999} \right) \\ &= \frac{15}{26} \end{aligned}$$